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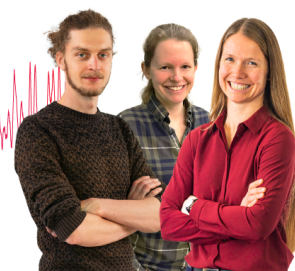
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Validation Typology of Middle School Students in Mathematical Modeling

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Abstract. This paper aims to describe the typology of student validation in mathematical modeling. The subjects of this research were 60 State Middle School students in Surabaya. Data collection through mathematical modeling tasks, validation sheets and interviews. Data analysis of the validation typology was guided by Czoher (2018). The results show that each type of validation contributes to students' answers. Each student validates the mathematical solution by checking the calculation operations carried out so that they come to the conclusion that the solution obtained is correct or requires correction. A total of 40 students validated the mathematical solutions obtained for the calculation operations carried out. Thirty-one students indicated that they had readjusted the mathematical model created to the situation model, and 32 students adjusted their mathematical model to the real model. However, validation of real results had the lowest score; 29 students verified the adequacy of the real results to the situation model, and only 20 students adjusted the real results to the real model. These results imply that students are able to carry out validation but need to improve in verifying real results in the context of the problem, so learning is needed to develop mathematical modeling skills by emphasizing this.

INTRODUCTION

Mathematical modeling is closely related to problem-solving because mathematical modeling aims to solve real context problems [1],[2]. Mathematical modeling drills the students to find the relationship between real context and mathematics so they are able to solve real context problems and obtain meaningful learning [3]. Mathematical modeling accommodates students' exploration of real-life situations based on mathematical concepts in mathematics learning. With the transition to real context situations with mathematics, mathematical modeling has a significant role in children's cognitive development [4]–[6]. Mathematical modeling supports students in identifying the role of mathematics in life and the ability to use mathematical knowledge in solving real context problems [7]. Mathematical modeling skills can measure students' readiness to solve problems encountered in the future [8].

The mathematical modeling process theory was put forward by several researchers [9]. In substance, the mathematical modeling process put forward is relatively the same. In general, the modeling process involves understanding the real context of the problem, designing a mathematical model, completing the model through mathematical operations, evaluating the results, and communicating the most appropriate results. There are two processes that are used to evaluate the results: interpretation and validation [9].

The validation stage aims to re-examine the solution process by adjusting the model. After interpreting the results, the mathematical modeling process will continue with activities to validate the results. The validation stage can influence or even change the model assumptions raised at the beginning of modeling [10]. If the solution obtained is not proven appropriate to the real context, then a review of the mathematical modeling process is needed. This results in the repetition of mathematical modeling at several stages or even at all modeling stages (if necessary).

Validation is an important stage in the mathematical modeling process. The results of the validation stage will determine the consideration of model modifications and repeating the modeling cycle [11]. Validation plays an important role in the iterative nature of mathematical modeling. Apart from that, validation also plays a role in assessing the solution, which results in acceptance, model revision, or rejection [12]. Therefore, students can find the right solution by considering all relevant aspects because of the validation stage. In recognizing and identifying the types of validation that emerge, there are five typologies of validation in mathematical modeling [13]. The validation typology is composed of interconnected validation objects and standards. Czoher [13] validation objects come from

mathematical results, mathematical expressions, and real results. Meanwhile, validation standards are a reference for conformity to the final results of the validation object. Based on Czochoer's [13] validation typology, it can be concluded that each final result at each stage of the mathematical modeling process can be validated by checking its suitability for the previous stages.

In fact, the important validation stage in mathematical modeling receives little attention in mathematics learning. Model validation is something that teachers often ignore by not accustoming students to carrying out these activities in learning [14]. The textbooks used usually do not raise real context problems that can be used in implementing the mathematical modeling stages as a whole. In addition, mathematical modeling requires much time for teachers and students [1]. This makes schools pay less attention to mathematical modeling activities, especially at mathematical modeling stages such as validation.

Several previous research studies on mathematical modeling have proven that the validation stage is one of the stages of mathematical modeling that students master [15]. The research they conducted focused on students' mathematical modeling process in solving real context problems. Some of these studies describe the mathematical modeling process as a whole without referring specifically to one or several stages in the modeling. These studies agree that the validation stage is a student's weakness in every modeling process.

The validation stage is important in mathematical modeling based on the description above. Several previous research on mathematical modeling focused on comprehensively uncovering the mathematical modeling process of middle school students [11]. Specific research related to the validation stages in the modeling process is important for mapping validation descriptions. of the typology validation carried out. In the future, this can be continued with determining the appropriate type of treatment that must be given to familiarize students with carrying out validation at each stage of mathematical modeling.

METHOD

This research is descriptive with a qualitative approach that aims to describe the typology of student validation in mathematical modeling. A qualitative approach is a means of exploring and understanding a phenomenon in groups and individuals [16]. Meanwhile, descriptive research aims to provide detailed, systematic, factual, and accurate descriptions. This research describes a typology of student validation in mathematical modeling based on the typology of validation stages developed by Czochoer[13]. In the validation typology developed, Czochoer [13] divides validation activities into five types. In general, there are activities to validate math results, math expressions, and real results, which are reviewed based on the situation model and real model. Meanwhile, the instruments for this research are mathematical modeling tasks, validation sheets, and interviews.

The subjects of this research were 60 State Middle School students in Surabaya. The selection of research subjects used purposive sampling based on the consideration of looking for samples with special requirements to achieve research objectives. The special requirement referred to in this research is that students provide various types of validation on the validation sheet when completing the Mathematical Modeling Task (MMT). In addition, to obtain a description of the mathematical modeling process, subjects will be selected based on completeness in MMT.

In general, research data was analyzed based on Czochoer's [13] typology of validation stages. In the validation stage, there are a series of systematic steps to consider the adequacy of a solution that has been obtained by adjusting the situation model. For this reason, it is possible to have various results from the student validation stages with various adjustments that they design. Based on this description, semi-structured interviews are suitable for digging deeper into the validation stages of students' mathematical modeling. Semi-structured interviews are situations where researchers can develop interview questions while still being guided by interview guidelines [16].

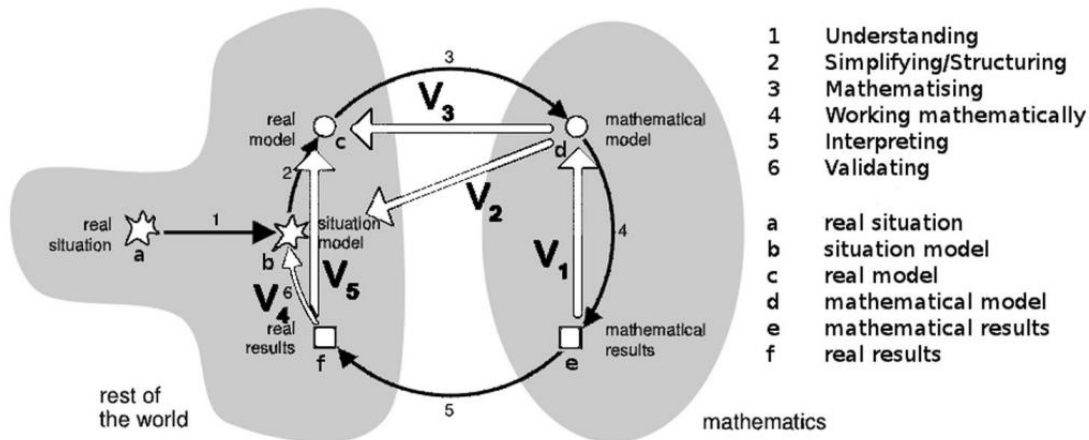


FIGURE 1. Validating Typology in Mathematical Modeling by Czoher

The first author discussed and validated the validation process typology used in this study. Below is a validation process typology that classifies various types of validation processes during the mathematical modeling process.

TABLE 1. Description Validating Typology

Type	Validation Object	Standard Validation	Deskripsi
V_1	Mathematical Result	Mathematical Expression	Examine the result of working mathematically or analyze the mathematical work process.
V_2	Mathematical Expression	Situation Model	Comparing mathematical models, components, and relationship of solution factors with situation model
V_3	Mathematical Expression	Real Model	Comparing the mathematical models, components, and relationship of solution factors with the ideal version of the real model
V_4	Real Result	Situation Model	Comparing real results with situation models or empirical data (predictions by particular theory)
V_5	Real Result	Real Model	Comparing real results with the principles used in the ideal version of the real model

RESULT AND DISCUSSION

Based on validation sheets from 60 students, 152 types of validation emerged in completing the mathematical modeling test. The following is the frequency percentage of each type of validation from 152 validations based on Czoher's [13] mathematical modeling validation typology.

TABLE 2. Percentage of Validation Types from 60 Students

Tugas	V1	V2	V3	V4	V5
Mathematical Modelling Task	26 %	20 %	21 %	19 %	13 %

The results of the validation sheets from 60 students imply that >25% of the validation carried out by students was an examination of the mathematical solutions obtained by checking the correctness of the calculation operations that had been carried out. V_1 results confirm students' decisions regarding the mathematical solutions obtained. V_2 and V_3 have values that are not much different, implying that students have verified the mathematical expressions used in finding solutions by adjusting them to the situation and real models. V_4 and V_5 contribute to validating real results that can be adapted to the context of the problem. This type of validation only appears in no more than 20%, which indicates that students do not validate the real results obtained. Further, each type of validation carried out by students will be described.

First Type Validation (V_1)

The first type of validation (V_1) plays a role in verifying mathematical solutions by checking the correctness of the mathematical calculation operations that have been carried out. V_1 is intended to ensure the correctness of the mathematical solution or whether changes and recalculations are needed. It is not uncommon for V_1 to cause students to change the mathematical solution that was originally found. Rianti shows type 1 validation in the solution process until a decision change occurs in determining the correct mathematical solution.

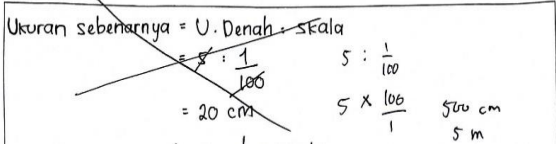
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FIGURE 2. Rianti Recalculation

It is clear that at the beginning of the completion process, Rianti crossed out several calculation steps to find the actual length of the building. Apart from that, it is also implied that Rianti did V_1 by checking again and recalculating on the right side. After checking and recalculating, Rianti changed her answer in the following way.

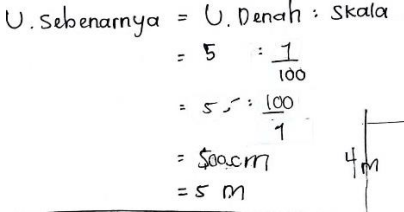
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FIGURE 3. Changes in the Method of Calculating Rianti

Rianti realized that the mathematical calculation operation carried out previously was wrong, so she changed her answer. In this case, V_1 plays a role in ensuring the correctness of the mathematical solution by checking and recalculating carried out by Rianti. This is also reinforced by Rianti's answer in the validation sheet, which stated that she had double-checked the formula used and ensured the correct use of the numbers.

1. Dalam memastikan bahwa solusi matematis yang kamu temukan itu benar, apa yang kamu lakukan? mengecek ulang rumus yang saya gunakan, memastikan angka yang saya gunakan tidak tertukar	Translation double check the formula I used, making sure the numbers I used are not mixed up
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FIGURE 4. Type 1 Validation Sheet Answers

Second Type Validation (V_2)

V_2 works in comparing mathematical expressions to the model situations created. This type of validation ensures the suitability of the conditions of the problem context with the mathematical expression chosen in finding a solution to the problem. V_2 accounted for 20 % of the 152 validations that appeared. The results of this validation will provide considerations regarding changes to assumptions, variables, relationships, and conditions if needed to modify the model [13].

Bellinda compares the suitability of the mathematical expressions used to the model situation according to the existing problem. The situation model represents the floor plan of a shop, and it ensures that the mathematical expression is correct at the start of the solution, i.e., it uses the concept of scale to find the actual size of the shop.

Bellinda: I confirm the method I use by returning to the problem. At first, I adjusted the information regarding the shop's floor plan to be built; I ensured that the correct scale concept was used in the first step in solving this problem. For the area concept calculation operation, I also ensured that the context of this shop problem discussed walls that appeared to be rectangular.

This statement can be strengthened by detailed small notes that are repeated on the question sheet as shown in Figure 5.

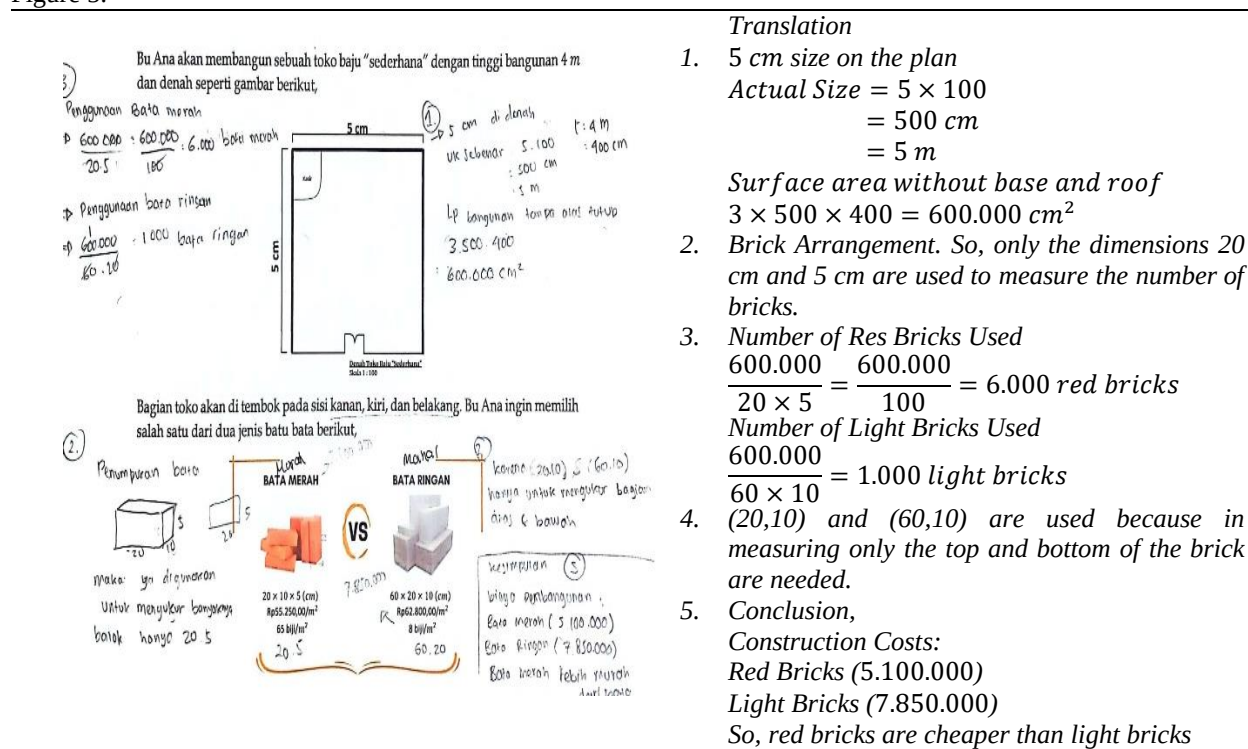


FIGURE 5. Bellinda Validation Activity

Third Type Validation (V_3)

The third type of validation comes from the same validation object as V_2 , namely the mathematical expressions selected and used in the process of solving mathematical modeling problem tests. V_3 accounts for 21% of the 152 validations available. The validation of mathematical expressions in V_3 is adjusted to the real model that the modeler built. Real models provide limitations and various assumptions that allow them to be effectively used in answering the problems presented. Checking mathematical expressions against the limits of the model being built is very important in ensuring that the mathematical expressions used are appropriate and sufficient to be used in the problem-solving process. Not only assessing the adequacy of the mathematical expressions used, V_3 also contributes to raising awareness of changes in model assumptions in finding the best solution [13]. The following interview shows this.

Researcher: What if the initial conditions shift, initially asked about the minimum cost of purchasing bricks for three sides of the wall. Now, two sides of the shop are covered with glass while only two sides are walled. How to solve it?

Caca: Use the same method, only different in that the number of sides of the wall is changed to multiplied by 3 to be multiplied by 2

Students are aware of the differences in situations mentioned by researchers. Students can assume changes in input values in the calculation process without carrying out the calculation process empirically. However, students can still make decisions regarding the effect of changes in the input values mentioned. Students realize that the mathematical solution will change as the input value changes in the specified mathematical expression. V_3 plays a role in considering the effect of changes in input values in mathematical expressions on the final solution obtained.

Fourth Type Validation (V_4)

V_4 involves the modeler's interpretation of the final results and adapting them to real context situations. This type of validation accounted for 19 % of the 152 validations that appeared. In this validation, considerations arise regarding the influence of aspects of the real context in the situation model on the final solution. V_4 also raises considerations regarding the relationship of the mathematical solution with the situation model to the modeler's final solution.

Abyas explained that he gave an answer that shifted from the final results he obtained because there were several considerations related to the model situation. Abyas changed his decision to provide an answer because he adapted the solution obtained to the context of real-life problems. He considers other factors that are not in the question. He felt that these real context factors greatly influenced his final solution.

Abyas: I still prefer light bricks when building a shop even though my calculations show that red bricks are cheaper. It is quicker to set light bricks than red bricks, but the price difference is small. I still decided to use light bricks.

Abyas considered other factors not listed in the question based on his personal experience in everyday life. Abyas also made adjustments to the mathematical solution obtained to ensure that the final solution was chosen based on consideration of the model situation built at the beginning of modeling. Maaß (2006) emphasized that in indicating validation there is consideration regarding the influence of aspects of the real context on the solution found. In addition, students considering the influence of the relationship of the mathematical solution with the situation model emphasize that the relationship strengthens the stated final solution. This is in accordance with the opinion of Czocher (2018), who states that in validation, there is an activity of considering the influence of other aspects in the calculation process without carrying out an empirical recalculation process. In this case, V_4 plays a role in changing the final result obtained by adapting it to the model situation and problem context.

Fifth Type Validation (V_5)

Of the 152 existing validations, V_5 contributed the least validation activity, namely 13%. Similar to V_4 , V_5 contributes to validating the final solution found by the modeler. However, V_5 is not adapted to the model situation but to the real model. V_5 plays an important role in the nature of different modeling iterations as many times as deemed necessary for students to obtain satisfactory models and results [17]. Czocher [13] states that this kind of validation contributes to the modeler's considerations regarding the final solution obtained. By changing the constraints or input values in the calculation process, students are able to evaluate the possible consequences of the final solution. The following are activity V_5 , which causes students to change the entire solution process carried out because the final results obtained are insufficient to answer the question.

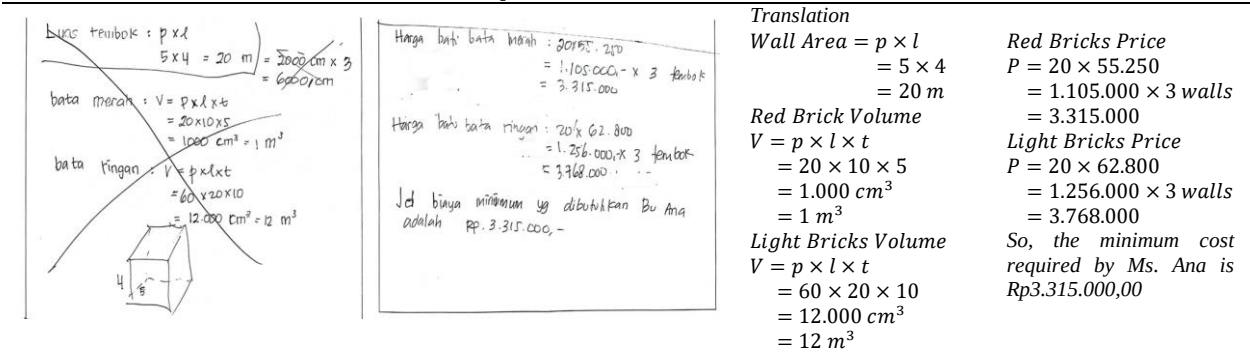


FIGURE 5. Changes to all Models Due to V_5

Muhaldin considered that more than the final results found in the initial settlement process were needed to answer the existing problems. He realized there were errors in the selection and use of mathematical concepts in the solution process. So, he decided to repeat the calculation process, starting with changing the mathematical concepts used. In this case, it can be concluded that validation is a benchmark that shows that the model needs improvement [18]. Suppose the solution or mathematical modeling process that has been carried out needs to be proven to be appropriate to the real context situation. In that case, it is necessary to repeat certain stages or even all stages of mathematical modeling [9].

The variety of validations carried out by students implies that validation has a significant role in the mathematical modeling process. Validation is the key to the iterative nature of the mathematical modeling process, which indicates checking or repetition in part or all of the mathematical modeling process [19]. V_1 plays a role in checking the correctness of the mathematical solution obtained by confirming the mathematical calculation operations that have been carried out or by recalculating. Based on the mathematical modeling process, V_1 will result in continued validation, namely verification of the mathematical calculation operations used with adjustments to the situation model and real model, which in the validation typology are then called V_2 and V_3 . The continuity of V_1 , V_2 , and V_3 can be seen from the validation results that appear, and it is stated that V_1 , V_2 , and V_3 each contribute >20% of the total

validation that appears. In contrast to V_1 , V_2 , and V_3 , which are mutually continuous, V_4 and V_5 build new validation objects that start from verifying the final results found. V_4 and V_5 check for the final results of the mathematical modeling process to be adjusted to the situation model and real model, resulting in a decision on the most effective and appropriate solution for the modeler.

CONCLUSION

Based on the results and discussion of research data related to the description of validation typologies for junior high school students in mathematical modeling, it is proven that each validation contributes to the mathematical modeling process. In completing the mathematical modeling test, 152 validations were obtained on the student validation sheet. As many as 26% were in the first validation type category (V_1) by confirming the mathematical solution obtained based on checking the correctness of the mathematical calculation operations that had been carried out. This check is carried out by ensuring the correctness of mathematical concepts, the correctness of each number used, and recalculation if necessary. V_2 and V_3 show similar results, with percentages of 20% and 21%, respectively. Students have adapted the mathematical expressions used to the situation model and real model. V_2 is proven to play a role in checking the adequacy of the mathematical expression used based on the problem model situation. At the same time, V_3 ensures that the mathematical expression is in accordance with the limits of the model being built or is successful in considering the effect of changes in model limits on the mathematical expression. Even though each validation contributes to students' answers, in fact students pay less attention to validation regarding the final results obtained. This can be seen in the results of V_4 , which only contributed 19%, and V_5 , 13% of the total student validation that appeared. Therefore, further research is needed to discuss the low level of validation of students' final results in the mathematical modeling process. Then, to improve mathematical modeling skills in the classroom, teachers need to emphasize students' skills in validating the final results in the solutions they carry out.

REFERENCES

1. D.H. Tong, N.P. Loc, B.P. Uyen, and L.T. Giang, [International Journal of Learning, Teaching and Educational Research](#) **18**, 18 (2019).
2. B. Riyanto, R.I.I.P. Zulkardi, and Darmawijoyo, in *Journal of Physics: Conference Series* (Institute of Physics Publishing, 2018).
3. R.B. Ferri and W. Blum, In Proceedings of the sixth Congress of the European Society for Research in Mathematics Education (pp. 2046-2055) (2010).
4. M.V. Krutikhina, V.K. Vlasova, A.A. Galushkin, and A.A. Pavlushin, [Eurasia Journal of Mathematics, Science and Technology Education](#) **14**, 1305 (2018).
5. Ş. Deniz and G. Kurt, [Participatory Educational Research](#) **9**, 150 (2022).
6. Y.P. Chang, J. Krawitz, S. Schukajlow, and K.L. Yang, [ZDM - Mathematics Education](#) **52**, 59 (2020).
7. D. Thufaila and Sugiman, in *Journal of Physics: Conference Series* (Institute of Physics Publishing, 2019).
8. F. Fakhrunisa and A. Hashanah, in *Journal of Physics: Conference Series* (Institute of Physics Publishing, 2020).
9. K. Maaß, [ZDM](#) **38**, 113-142 (2006).
10. R.B. Ferri, *Learning How to Teach Mathematical Modeling in School and Teacher Education* (Springer International Publishing, 2017).
11. Gurjanow and Ludwig, in *Mathematical Modelling Education and Sense-Making* (Switzerland, 2020).
12. J.A. Czoher, [Journal of Mathematical Behavior](#) **45**, 78 (2017).
13. J.A. Czoher, [Educational Studies in Mathematics](#) **99**, 137 (2018).
14. A.S. Zeytun, B. Cetinkaya, and A.K. Erbas, *Eurasia Journal of Mathematics, Science and Technology Education* **13**, 691 (2017).
15. A.H. Rosyidi, L.F. Fadhillah, and S. Hartono, In International Joint Conference on Science and Engineering 2021 (IJCSE 2021) (pp. 409-416). Atlantis Press (2021).
16. J. Creswell, *Research Design; Qualitative, Quantitative, and Mixed Methods Approaches* (SAGE Publication, Thousand Oaks, 2017).
17. L. Albarracín, J. Ärleback, E. Civil, and N. Gorgorió, [Mathematics Enthusiast](#) **16**, 211 (2019).
18. V. Geiger, [Educational Studies in Mathematics](#) **109**, 313 (2021).
19. J.A. Czoher, [Math Think Learn](#) **18**, 77 (2016).